Initial Noise and Quiet Start (Q.S.) in Electromagnetic Particle Simulation Codes

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We present the results of applying first and second moment quiet start, as suggested by Gitomer (*Phys. Fluids* **14** (1971), 1591), to simulation of electromagnetic ion-cyclotron instability. It is found that proper use of the above-mentioned method is necessary when accurate evaluations of growth rates with relatively few numbers of simulation particles are desired. An initial noise reduction of more than two orders of magnitude is obtained. This is equivalent to a large increase in the number of particles used, thus resulting in a substantial computer time saving. While carefully paying attention to such side effects as the "multibeam" formation, the Q.S. method, with its easy general applicability to 1-d, 2-d, and 3-d velocity space dimensions, may be recommended as a reliable and powerful tool for routine usage in particle computer simulation of electromagnetic instabilities.

1. INTRODUCTION

As is well known simulation computer codes use, as a rule, relatively few particles, thus causing discrete effects such as fluctuations and collisions to be exaggerated. Usually, nonphysical noise resulting in this way may be unimportant if the physical instability under investigation is relatively strong, but it may partly or completely veil it if it is very weak.

In order to remedy the situation, several methods have been devised in the past with the purpose of reducing the initial noise to an acceptable level, while still using a manageable number of particles in the simulation. In Byers' method [1, 2], each particle is loaded uniformly in space with a velocity chosen from a finite set of "beam" velocities in such a way as to yield a particular distribution function, $f(v)$. In this method growth starting from machine roundoff level can be observed at precisely the linear Vlasov theory predictions, up to the saturated level, many orders of magnitude above the starting point.

Morse and Nielson [3] have used a first moment correction procedure of more general applicability to remove some of the noise inherent in a randomly generated 2-d Maxwellian initial velocity distribution. Their anisotropy is $A = T_\perp/T_\parallel - 1 = 24$, a rather large value, so that only first moment correction should be sufficient in revealing the physical instability.

118

Gitomer [4–6] used an extended version of Morse and Nielson's method in an electrostatic plasma code by applying first and second moment corrections to the velocity components. In his simulations he actually did not find a significant reduction of noise level in comparison with the nonquiet start runs.

In the 1-d electromagnetic code we have used, devised by Forslund *et al.* [7], the initial velocity components of consecutive particles are generated as (R_1 and R_2 being random numbers uniformly distributed over the range (0, 1), and v_{th} being the initial thermal velocity).

$$v_{x,y,z} = v_{th}^{x,y,z}(-\ln(1 - 0.999999 R_1))^{1/2} \cos(2\pi R_2).$$

Then, first moment correction on batches of 1000 particles each is routinely applied. We designate this original unchanged code as the non-Q.S. code. Q.S. will denote that both first and second moments are corrected, as suggested by Gitomer.

We used Gitomer's method in simulation of electromagnetic ion-cyclotron instability with considerable success. In one case [8], the anisotropy was $A = 4$, a moderate value, and the growth rates found from the simulation were very close to the linearly predicted values. A parallel non-Q.S. run gave growth rate values lower by a factor of four than the linearly predicted ones. In another case [9] the anisotropy was $A = 0.5$ and when no Q.S. was applied, the electromagnetic wave developing in the system was at all times completely screened by the noise level. When Q.S. was applied on every cell, the initial noise was reduced by more than two orders of magnitude, enabling accurate determination of the growth rates involved[1].

It is the purpose of this paper to present some quantitative comparative details about the initial conditions prevailing in Q.S. vs./non-Q.S. runs. We find that use of second moment Q.S., applied in a proper way and according to Gitomer's method, is generally recommendable and in some cases (weakly unstable plasmas) it is absolutely indispensable in electromagnetic particle simulations using relatively few particles. In some respects, the Q.S. method is found to be equivalent to a large increase in the number of particles used in the simulation, thus resulting in a substantial computer time saving.

2. SOME REMARKS ON INITIAL NOISE IN COMPUTER SIMULATION

Even in a simulation of a linearly stable plasma there is a slow development of electromagnetic waves, because of the small number of particles, i.e., a rather large

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[1] A possible explanation of the discrepancy between our large noise reduction and the insignificant noise reduction (several percents) in Gitomer's simulations may be provided by the fact that Gitomer used an electrostatic plasma code and the noise was measured using total electrostatic field energy as a diagnostic, while our code was electromagnetic and the electromagnetic field energy was taken as a measure for the noise produced in the system. Because the quiet start is applied to the velocity distribution function which is directly connected with the magnetic field energy (which is the main part of the e.m. energy, the electrostatic field being negligible in all our cases), we expect the electromagnetic ($\approx$ magnetic) noise in our cases to be much more sensitive to the quiet start smoothing than the electrostatic noise in Gitomer's cases.

initial noise (see Ref. [10, p. 101]; for illustration see Ref. [9, Fig. 4], in which the case $A = 0$ should be linearly stable).

Sagdeev and Galeev [11] pointed out that fluctuations in a plasma may be expected to provide an additional source term in the wave kinetic equations (see also Ref. [12]) which generate spontaneous plasma waves. This additional term is not significant in the case of strongly unstable plasmas. Nevertheless, in weakly unstable or in stable plasmas the competition between the fluctuation term and the physical instability source determines the equilibrium level of the thermal fluctuation.

For subsequent reference we present here a comparison of two simulations with different numbers of particles. In the first case the number of protons and electrons was 6000 and in the second case it was 18000. The thermal anisotropy of the protons was $A \equiv T_{\perp}/T_{\parallel} - 1 = 0.5$ in both cases, and all the other parameters were the same ($\Delta t = 13.55 \, \omega_{pe}^{-1}$; $\Delta x = 5.94 \, c\omega_{pe}^{-1}$; No. of cells = 100; $m_p/m_e = 10$; ω_{pe} = electron plasma frequency; c = light velocity; the non-Q.S. code was used in these cases). The results are summarized in Table I.

TABLE I

$A \equiv T_{\perp}/T_{\parallel} - 1 = 0.5^a$

No. of particles	$\dfrac{\Delta E^{TOT}}{E^{TOT}}$	$\dfrac{WB(0)}{E^{TOT}}$		$\left(\dfrac{T_{\perp}}{T_{\parallel}}\right)^p$		$\dfrac{WB(5\tau_p)}{WB(0)}$
	$t = 5\tau_p$	$t = 0$	$t = 5\tau_p$	$t = 0$	$t = 5\tau_p$	
6000	$-1.9 \cdot 10^{-5}$	$3 \cdot 10^{-3}$	$1 \cdot 10^{-2}$	1.501	1.394	3.0
18000	$1.8 \cdot 10^{-6}$	$1.5 \cdot 10^{-4}$	$0.4 \cdot 10^{-2}$	1.501	1.440	24.0

^{a} Notations: E^{TOT} = total energy in the system; WB = magnetic wave energy; $T_{\perp}$, $T_{\parallel}$ = temperatures perpendicular and parallel to the external magnetic field, respectively; τ_p = proton gyroperiod; $\Delta E^{TOT} = E^{TOT}(t) - E^{TOT}(0)$.

As can be seen, the result of increasing the number of particles by a factor of 3 is to reduce the relative initial noise level $WB(0)/E^{TOT}$, by a factor of 20. At $t = 5\tau_p$ this factor is only 2.5, indicating a much faster initial growth rate of the electromagnetic wave in the case with 18000 particles (also see the last column in Table I). Other features worth mentioning are the improvement in energy conservation $\Delta E^{TOT}/E^{TOT}$ (E^{TOT} being the total energy in the system), when more particles are used, and the faster relaxation of the anisotropy $T_{\perp}/T_{\parallel}$ ($\equiv A + 1$) when a smaller number of particles are used (see Refs. [10, 11]). As it will be seen later, the quiet start procedure of Gitomer [4–6] when applied to relatively small numbers of particles, causes similar improvements in the simulation results. One important difference is that whereas the Q.S. assures only initial low noise, the increase of the number of particles used results

in a persistent noise reduction. In general, however, application of Q.S. is recommendable (even in cases where increase in the number of particles is anyway required), in order to correctly compute linear growth rates.

3. TECHNICAL DETAILS OF THE Q.S. METHOD

The moment correction method of Gitomer [4–6] is intended to achieve local velocity distribution functions with zero first moments and correct second moments corresponding to the initial Maxwellian chosen.

The corrected velocity components of particle i, denoted by v_{xi}^*, v_{yi}^*, v_{zi}^* are obtained from the randomly generated values v_{xi}, v_{yi}, v_{zi}, in the following manner. A group of N adjacent simulation particles are considered.

The following quantities are computed

$$\bar{v}_x = (1/N) \sum_{i=1}^{N} v_{xi}; \qquad \bar{v}_y = (1/N) \sum_{i=1}^{N} v_{yi}; \qquad \bar{v}_z = (1/N) \sum_{i=1}^{N} v_{zi},$$

$$\bar{v}_x{}^2 = (1/N) \sum_{i=1}^{N} v_{xi}^2; \qquad \bar{v}_y{}^2 = (1/N) \sum_{i=1}^{N} v_{yi}^2; \qquad \bar{v}_z{}^2 = (1/N) \sum_{i=1}^{N} v_{zi}^2.$$

Then

$$v_{xi}^* = \alpha(v_{xi} - \bar{v}_x),$$
$$v_{yi}^* = \beta(v_{yi} - \bar{v}_y),$$
$$v_{zi}^* = \gamma(v_{zi} - \bar{v}_z),$$

where

$$\alpha^2 = (KT_x/m)[\bar{v}_x{}^2 - (\bar{v}_x)^2]^{-1},$$
$$\beta^2 = (KT_y/m)[\bar{v}_y{}^2 - (\bar{v}_y)^2]^{-1},$$
$$\gamma^2 = (KT_z/m)[\bar{v}_z{}^2 - (\bar{v}_z)^2]^{-1},$$

and T_x, T_y, T_z are the desired initial temperature components. The corrections are made for similar contiguous groups of N particles each, throughout the system.

A problem which often arises in simulations using quiet starts, is the multiple beam instability [13]. In Byers' method [1, 2] this is an inherent feature, because distribution functions are approximated by a multitude of carefully chosen velocity beams.

In Gitomer's method [4–6] the "multibeaming" eventually appears when too few particles per "correction" group are used.

One can make the multiple beam instability weaker by taking more beams or by randomizing them in a convenient manner. In Gitomer's method the multibeam formation can be avoided by taking not too few particles per "moment correction" group. Such a moderate smoothing was found to be quite sufficient in all cases investigated by us.

4. Q.S. VS. NON-Q.S. RESULTS

In computing theoretical linear growth rates of electromagnetic ion-cyclotron waves, we used the following dispersion relation for bi-Maxwellian velocity distribution function and parallel propagation along a static and homogeneous magnetic field $\mathbf{B}_0$ [8]

$$c^2k^2 = \omega^2 + \sum_j \omega_{pj}^2 \left\{ A_j - \frac{1}{2^{1/2}v_{j,\text{th},\parallel} \cdot k} Z(\xi) \left[(A_j + 1)(\Omega_j - \omega) - \Omega_j \right] \right\}$$

where the summation is over all (+) and (−) plasma components. The notations used are as follows: $\omega = \omega_r + i\omega_i$, complex frequency; k, real wavenumber of disturbance; ω_p, plasma frequency; c, light velocity; Ω_j, the cyclotron frequency defined by

$$\Omega_j = q_j \mathbf{B}_0 / m_j c$$

and $v_{\parallel,j,\text{th}} \equiv (KT_\parallel{}^j/m)^{1/2}$, the thermal velocity of species j in the direction parallel to $\mathbf{B}_0$. Other notations are

$$A_j \equiv (T_\perp/T_\parallel)_j - 1$$

and $Z(\xi)$, the plasma dispersion function defined by

$$Z(\xi) \equiv (1/\pi)^{1/2} \cdot \int_{-\infty}^{+\infty} \left[\exp(-t^2)/(t - \xi) \right] dt$$

with

$$\xi_j = \frac{1}{2^{1/2}} \frac{\omega - \Omega_j}{v_{j,\text{th},\parallel} \cdot k}.$$

For the purpose of comparison and relative generality, three cases of ion-cyclotron electromagnetic waves developing in anisotropic, uniform, infinite, and magnetized proton plasmas were considered.

(a) $A(\equiv T_\perp/T_\parallel - 1) = 0.5$; $\Delta t = 13.55 \; \omega_{\text{pe}}^{-1}$; $\Delta x = 5.94 \; c\omega_{\text{pe}}^{-1}$; No. of cells $=$ 100; $m_p/m_e = 10$.

(b) $A = 4$; $\Delta t = 13.55 \; \omega_{\text{pe}}^{-1}$; $\Delta x = 1.3 \; c\omega_{\text{pe}}^{-1}$; No. of cells $=$ 100; $m_p/m_e = 10$.

(c) $A = 99$; $\Delta t = 1.355 \; \omega_{\text{pe}}^{-1}$; $\Delta x = 0.20106 \; c\omega_{\text{pe}}^{-1}$; No. of cells $=$ 100; $m_p/m_e =$ 10.

They represent weakly, moderately, and strongly unstable plasmas, respectively. Short initial periods were investigated. Some representative results of the simulation are summarized in Table II, and in Figs. 2 and 3.

We considered Q.S. over groups of different numbers of particles. In the $A = 0.5$ case with no Q.S. applied, the instability, represented by WB (magnetic wave energy) is completely screened at all times by the large noise level. When $\langle \text{Q.S.} \rangle_{20}$ (Q.S. on groups of 20 protons) is applied, we get from the simulation an effective growth rate

TABLE II

Simulation Results (5000 Protons, 5000 Electrons, 100 Cells)[a]

Desired initial anisotropy $A \equiv (T_\perp/T_\parallel) - 1$	Q.S. status	$\Delta E^{TOT}/E^{TOT}$		WB (magnetic wave energy)			$T_\perp/T_\parallel \ (\equiv A + 1)$			$\dfrac{WB(t_1)}{WB(0)}$	E^{TOT} (total energy)
		t_1	$0.3\,\tau_p$	$t = 0$	t_1	$0.3\,\tau_p$	$t = 0$	t_1	$0.3\,\tau_p$		
0.5	no Q.S.	−2.9-5		6.7-3	6.7-3	6.7-3	1.52	1.52		1.00	1.35-1
	⟨50⟩	−1.8-6		6.3-5	6.6-5		1.50	1.50		1.05	1.35-1
	⟨20⟩	3.-6		1.8-5	2.8-5		1.50	1.50		1.55	1.35-1
	⟨2⟩	−4.3-7		2.4-7	2.7-5		1.50	1.50		112	1.35-1
4	no Q.S.	−1.8-5		3.-3	3.-3		5.08	5.07		1.00	1.90-1
	⟨50⟩	−2.7-6		2.8-5	3.1-5		5.02	5.03		1.10	1.90-1
	⟨20⟩	−9.8-8		7.9-6	1.3-5		5.02	5.02		1.65	1.90-1
99	no Q.S.	−9.3-5	8.9-4	2.1-4	7.5-3	1.4-2	101.2	20.6	11.2	35.7	5.4-1
	⟨50⟩	1.2-5	4.9-3	1.3-6	1.2-3	9.7-3	100.0	65.0	15.5	924.0	5.4-1

[a] Notations: ⟨50⟩ means that Q.S. is applied on groups of 50 protons each, etc. $t_1 = 0.05\,\tau_p$ for $A = 0.5$ case; $0.02\,\tau_p$ for $A = 4$ case; and $0.15\,\tau_p$ for $A = 99$ case. $-2.9\text{-}5$ means -2.9×10^{-5}, etc. Other notations are as in Table I.
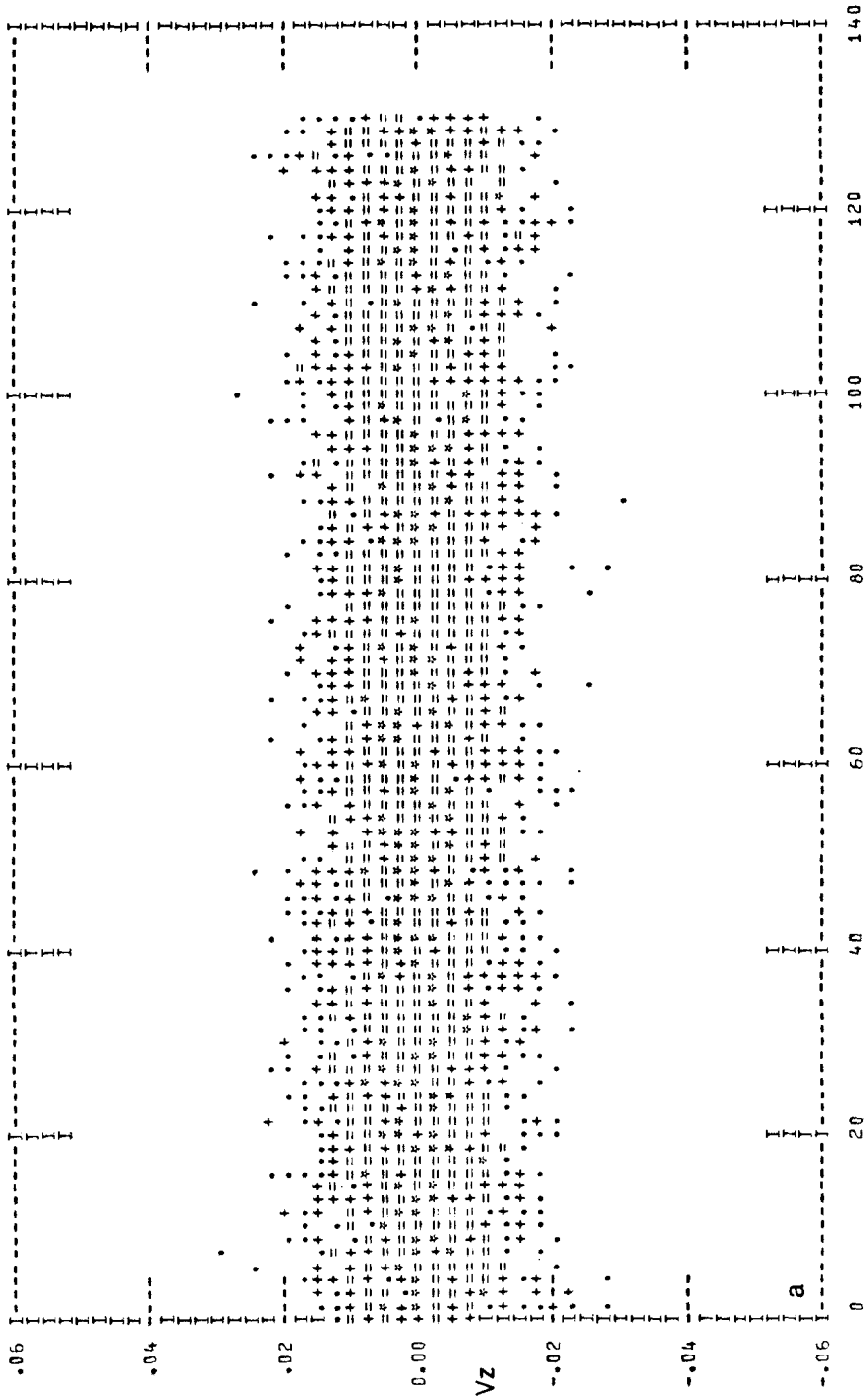
ION   TIME =   1.355

FIGURE 1a. $A = 4$, no Q.S.

FIGS. 1a–e.   Initial phase space plots for the case $A = 4$, describing v_z vs x in a grid of 5×100 cells. The signs are as follows: $\cdot$, 1 particle; $+$, 2–3; $=$, 4–7; $*$, 8–15; $\times$, 16–31. v_z is normalized to the light speed, c. x is in units of $c\omega_{pe}^{-1}$ (ω_{pe} is the electron plasma frequency).
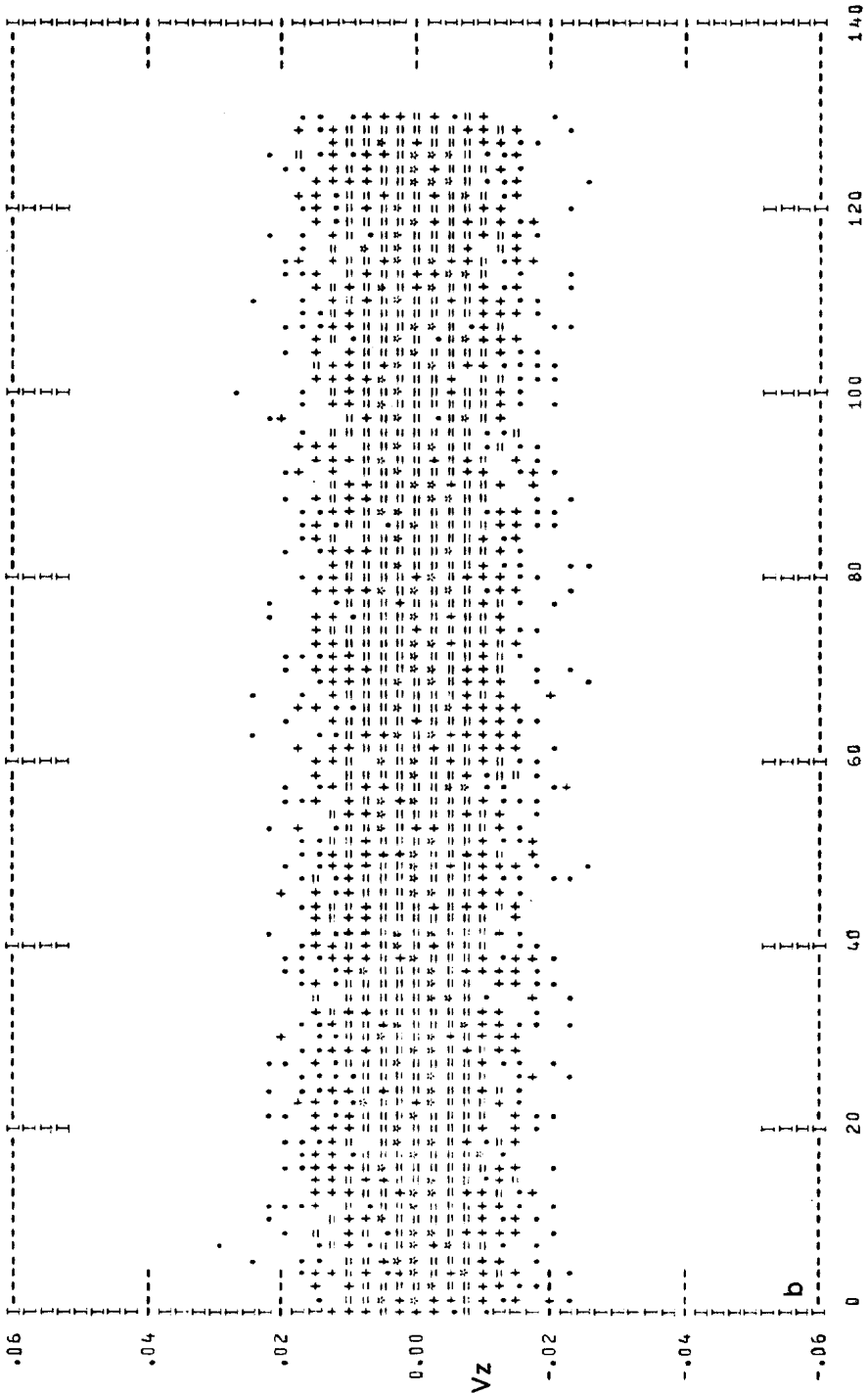
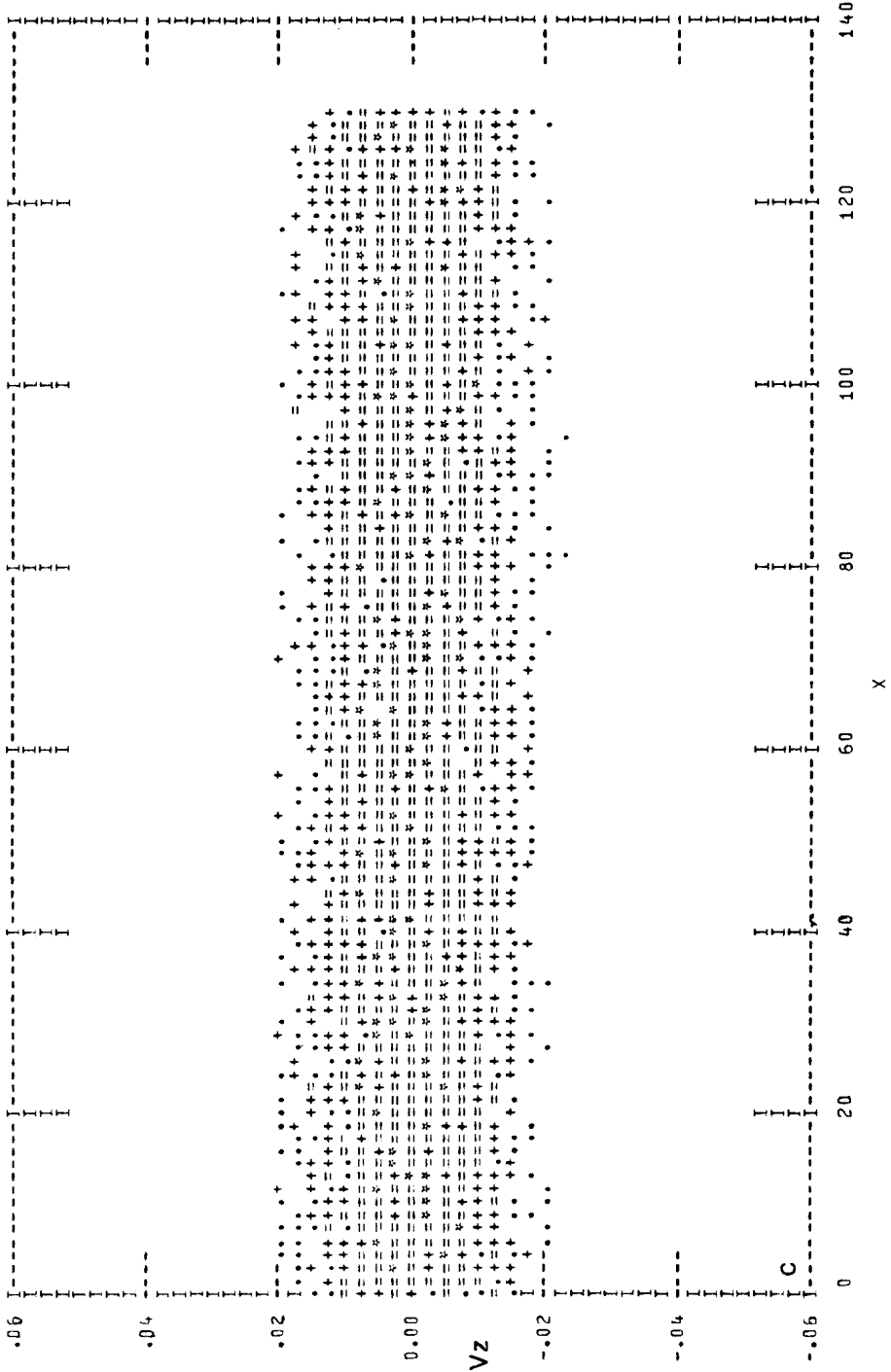FIGURE 1b. $A = 4$, $\langle Q.S. \rangle_{50}$.

ION   TIME =   1.355

FIGURE 1c. $A = 4$, $\langle \text{Q.S.} \rangle_{10}$.
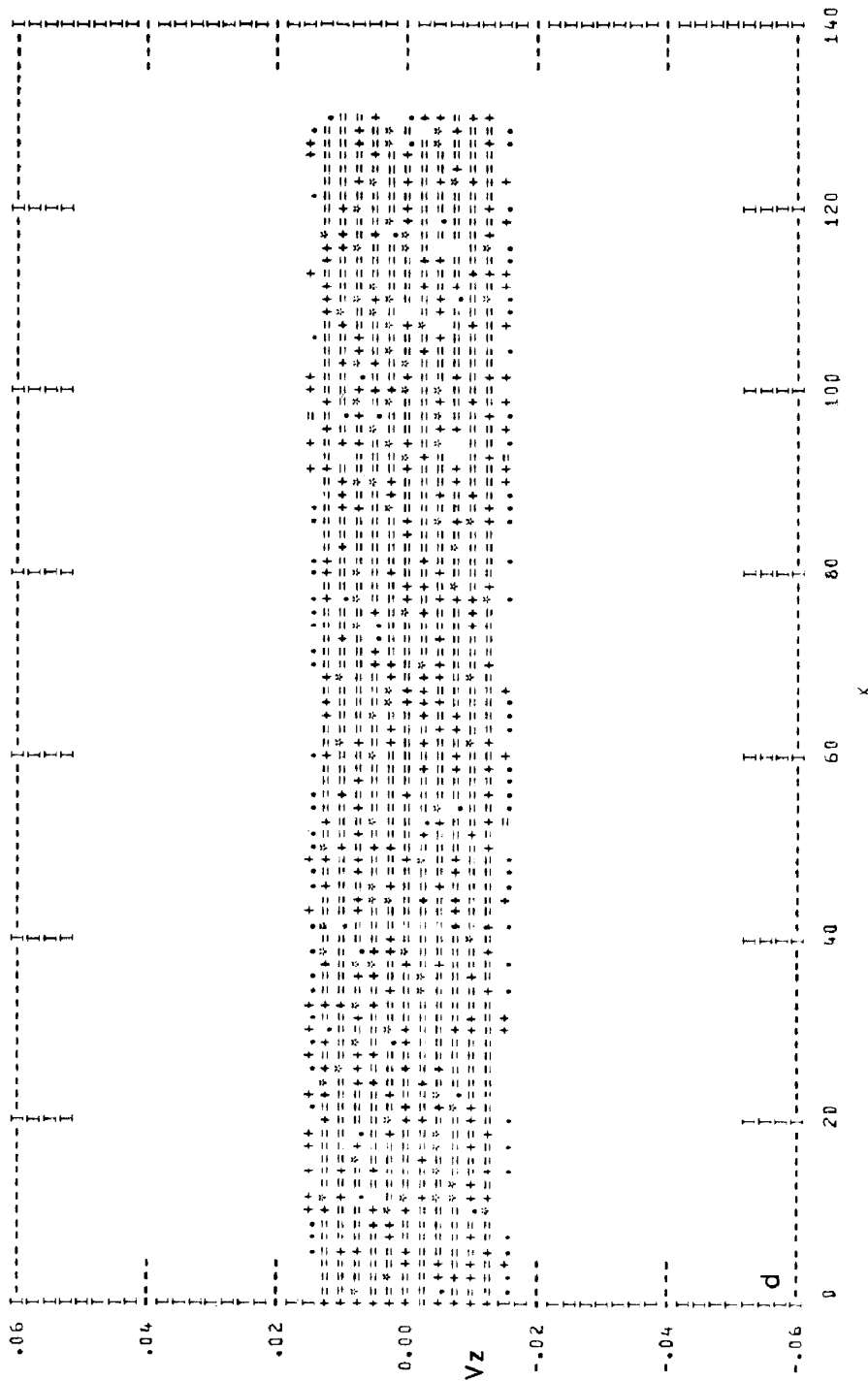
ION   TIME =   1.355

FIGURE 1d. $A = 4$, $\langle Q.S. \rangle_4$.

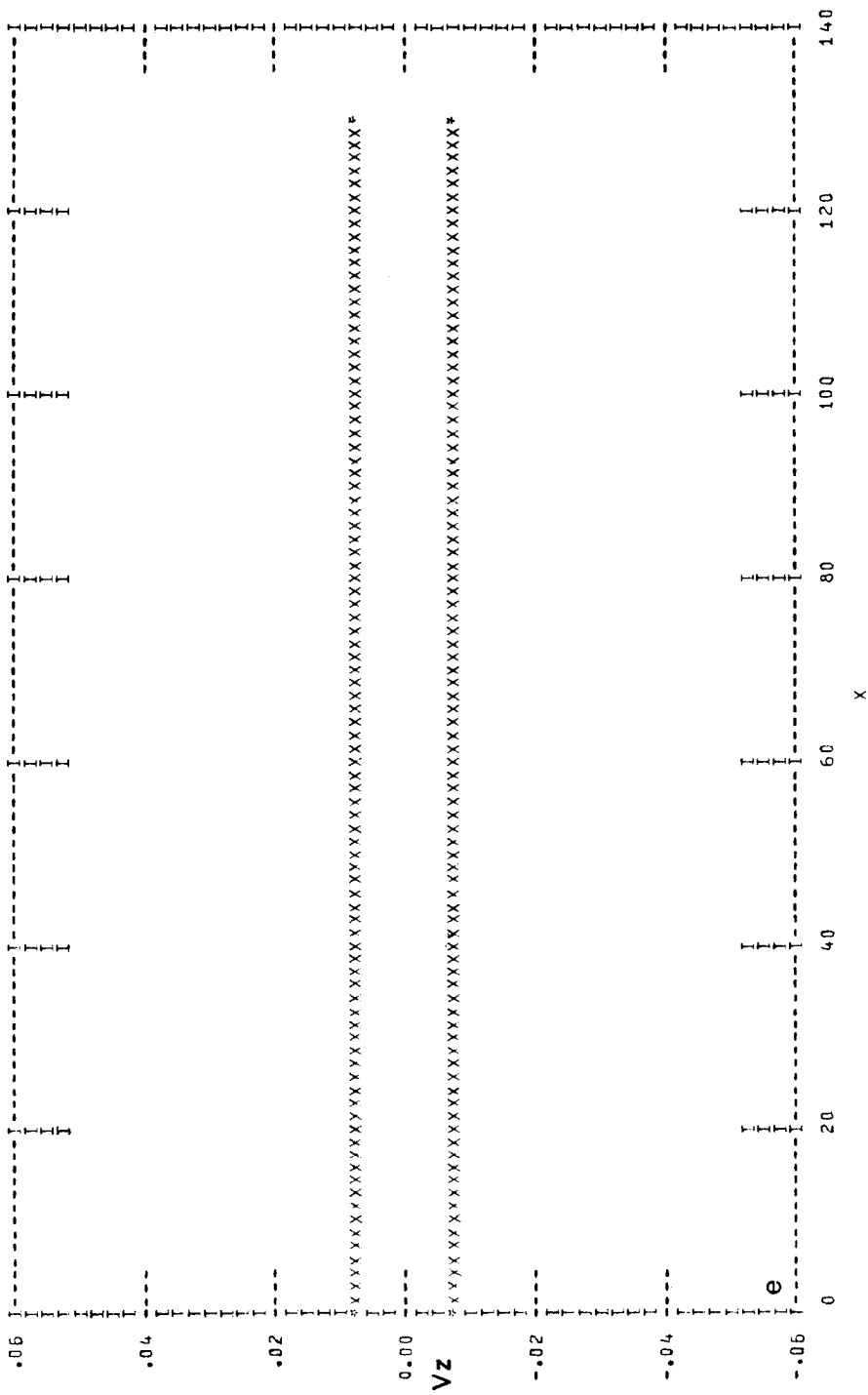ION   TIME =   1.355

Vz

Figure 1e. $A = 4$, $\langle Q.S. \rangle_2$.

$\gamma_{\text{eff}}^2 = 3 \times 10^{-3}$, which is in good agreement with the maximum linearly predicted value [9]. γ_{eff} is defined by the relation

$$WB(t_{\text{sat}}) = WB(0) \exp(2\gamma_{\text{eff}} \cdot t_{\text{sat}}),$$

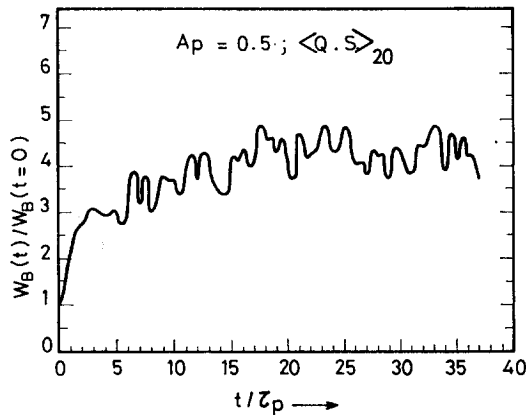t_{sat} being the saturation time of $WB(t)$.

FIG. 2. The temporal behavior of the normalized magnetic wave energy for the case indicated in the figure.
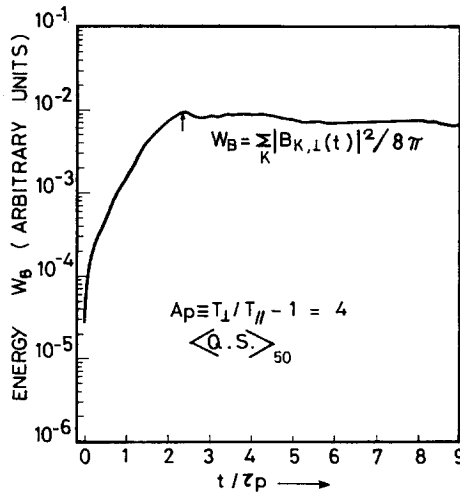

FIG. 3. The temporal behavior of the magnetic wave energy for the case indicated in the figure.

[2] All γ's in this paper are in units of Ω_p ($\gamma \equiv \text{Im } \omega/\Omega_p$).

In the $A = 4$ case, in the absence of Q.S., one obtains $\gamma_{\text{eff}} = 0.05$. When $\langle \text{Q.S.} \rangle_{50}$ is applied, $\gamma_{\text{eff}} = 0.2$, which is more compatible[3] with the linear prediction $\gamma_{\text{max}} = 0.4$ [8]. A Fourier analysis reveals that the growth rates of the individual modes, $\gamma(k)$, are in excellent agreement with the linear theory [8]. For convenience, the results are summarized in Table III.

TABLE III

Comparison of Observed and Linearly Predicted Growth Rates,
$\gamma(k)$, for $A = 4$ Case, with $\langle \text{Q.S.} \rangle_{50}$

Mode	Linearly predicted $\gamma(k)$	Observed $\gamma(k)$
1	~ 0	0.07
2	0.16	0.15
3	0.28	0.24
4	0.35	0.34
5	0.40	0.38
6	0.40	0.37
7	0.38	0.44
8	0.33	0.28
9	0.28	0.31
10	0.20	0.18
11	0.18	0.15
$k > 11$	~ 0	~ 0

Several obvious conclusions valid for all cases can be drawn from the results presented in Table II.

(a)   The initial anisotropy in the simulation is closer to the desired value if Q.S. is applied.

(b)   In initial stages isothermalization is faster when Q.S. is not applied, i.e., when initial noise is higher (eventually the anisotropies for Q.S. and non-Q.S. runs approach similar values at later times; see for example the $A = 99$ case), in agreement with our discussion in Section 2.

(c)   In all cases, the initial electromagnetic level $WB(0)$, is at least by two orders of magnitude lower when Q.S. is applied, than in absence of Q.S., and it grows faster in the initial stage, eventually approaching non-Q.S. values at later times.

(d)   Total energy conservation $\Delta E^{\text{TOT}}/E^{\text{TOT}}$, is not necessarily better when Q.S. is applied (see the case $A = 99$, at $t = 0.3\tau_p$).

(e)   Q.S. over smaller groups of particles gives still lower initial noise.

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[3] Different modes start with different initial amplitudes. They also start and saturate at different times, so it may happen that modes k with higher $\gamma(k)$ are smaller in amplitude (and vice versa) and this affects the γ_{eff} (overall growth rate) as calculated from $WB(t) = \sum_k B_k^2(t)$ or from weighting of $\gamma(k)$'s over all k-modes and over time.

However, as one can see from the phase space plots shown in Figs. 1a–1e and from the x-integrated v_z-distribution functions in Figs. 4a–4e, there is some minimal number of particles per group over which Q.S. is to be performed in order to avoid multibeam formation, this number being dependent on the simulation parameters of each case. A rough criterion for this minimal group of particles may be the condition that the distribution functions should not develop prominent double or multiple humps, when Q.S. is applied on them. This condition can be checked by a trial and error procedure, from a series of graphs like those shown in Figs. 1a–1e and 4a–4e. In our $A = 4$ case, for example, the initially Maxwellian distribution function is deteriorating badly somewhere between $\langle Q.S.\rangle_{10}$ and $\langle Q.S.\rangle_4$. In this case, we used $\langle Q.S.\rangle_{50}$ and found it to be sufficient for satisfactory evaluation of the growth rate (which is not too small). However, in the case of weak instability (e.g., $A = 0.5$), there is some contradiction between the requirement of low initial noise (which means smaller groups of particles for momentum correction) and the prevention of multibeam formation (which means not too few particles per group). We found the value of 20 particles per group to be satisfactory.
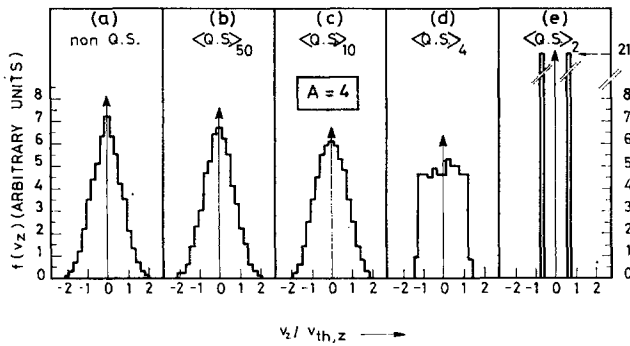
FIGS. 4a–e. Initial x-integrated v_z-distribution functions for the case $A = 4$ (corresponding to the phase space plots in Figs. 1a–e).

In conclusion of this section it can be said that application of Q.S. in a controlled manner is generally necessary in simulation of any anisotropically unstable plasma, if more precise evaluation of growth rates is desired. In this respect, the advantage of the Q.S. is that the system is perturbed at very low energy so that at saturation time the only large amplitude modes present are those with large linear growth rates [8].

While the present paper deals with unstable proton plasmas, it would be natural to extend the use of the Q.S. method to simulations of electron unstable plasmas (e.g., whistlers).

5. CONCLUSIONS

We used the Q.S. devised by Gitomer [4–6] in an electromagnetic 1-d code and found it very useful and in fact indispensable when weakly unstable plasmas [9] were

considered. In simulations of moderately [8] and strongly [14] unstable plasmas, the Q.S. method proved to be necessary for correct evaluation of linear growth rates of the electromagnetic instabilities.

The most important effect of using Q.S. is a substantial reduction by more than two orders of magnitude, of the initial noise. This means that the Q.S. procedure allows a relatively small number of simulation particles to behave for a finite amount of time nearly like a continuous system, thus offering a more accurate picture of the linear evolution. In this respect the Q.S. method is equivalent to a great increase in the number of simulation particles, thus resulting in exclusion of the fluctuational term as an additional nonphysical instability source [11]. The net result is a considerable computer time saving.

While carefully considering such side effects as the "multibeaming" process, the Q.S. method, with its easy general applicability to 1-d, 2-d, and 3-d velocity space dimensions, may be recommended as a reliable and powerful tool for routine usage in particle computer simulations of electromagnetic instabilities.

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